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IDMA vs. CDMA: Detectors, Performance and Complexity

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Abstract—This paper presents comprehensive comparisons of interleave division multiple access (IDMA) and direct sequence code division multiple access (DS-CDMA) in terms of performance and complexity using iterative multiuser detection technique, where we restrict ourself to three suboptimum linear detectors: minimum mean square error (MMSE), rake (or matched filter), and soft-rake detectors from practical concerns. We first analytically compare these detectors, which are found to be equivalent for IDMA with asynchronous users on flat channels, whereas this does not hold for DS-CDMA, which is sensitive to user asynchronism. This implies that, on flat channels, simple detector suffices to get the MMSE output for IDMA while DS-CDMA requires more complexity such as computing matrix inversions. We also discuss several aspects of complexities for IDMA and DS-CDMA when the MMSE detector is used. Computer simulations are performed in various scenarios and the performance is analyzed by bit error rate as well as by extrinsic information transfer chart. The analysis reveals some advantages of IDMA over DS-CDMA, particularly in highly user loaded scenarios.

I. INTRODUCTION

Interleave division multiple access (IDMA) is a recently proposed spread spectrum multiple access scheme similar to direct sequence code division multiple access (DS-CDMA or CDMA for short). Several techniques for IDMA have been studied in a number of literatures in the last years to further enhance the capability of IDMA. However, only a few papers have been concerned with comparisons of CDMA and IDMA systems so far. In [1], [2], only an uncoded system is compared. Although the comparison itself is interesting, uncoded systems are extremely unfavorable for CDMA systems as we will also discuss in this paper. The authors in [3] compare a coded system, but only the so-called soft rake detection is used for both CDMA and IDMA that is, again, unfavorable for CDMA systems. Another paper [4] compares uncoded and coded systems where the minimum mean square error (MMSE) detector is utilized for CDMA systems. That is a more suitable comparison, but the results are limited to one particular scenario. In any of the above mentioned references, the main focus is not the comparison of CDMA and IDMA. The first paper, which focuses on the comparisons of CDMA and IDMA, appeared in [5]. In this paper, we present more comprehensive comparisons that include different choices of detectors, analytical comparisons of the detectors, complexity analysis, and extrinsic information transfer chart analysis [6].

II. SYSTEM MODEL

Fig. 1 illustrates our general transmitter structure, which can be seamlessly used for both CDMA and IDMA systems. At the transmitter, information bits \(b_{i,k}\) of user \(k\), \(i \in \{1, \ldots, K\}\), are encoded by a rate \(R\) encoder. Low rate code is typically realized by a rate \(R_c\) convolutional code followed by a rate \(R_t\) repetition code. Then, it follows that 
\[
R = R_c R_t.
\]
The resulting code bits \(c_{j,k}, j \in \{1, \ldots, N_c\}\), are interleaved by a user-distinct bit interleaver \(\Pi_k\) to get \(c_{m,n,k}\), which are then mapped onto complex symbols \(x_{n,k}\) that are taken from a \(2^{N_m}\)-ary symbol alphabet: \(A \triangleq \{a_1, \ldots, a_{2^{N_m}}\}\), where we assume \(A\) to fulfill the following conditions: 
\[
\sum_{i=1}^{2^{N_m}} a_i = 0 \quad \text{and} \quad 2^{-N_m} \sum_{i=1}^{2^{N_m}} |a_i|^2 = 1.
\]
Note that \(c_{m,n,k}\), \(m \in \{1, \ldots, N_m\}\), \(n \in \{1, \ldots, N_c\}\), denotes the \(m\)-th bit of the \(n\)-th transmit symbol \(x_{n,k}\) of user \(k\), i.e., code bits \(c_{1,n,k}, \ldots, c_{N_m,n,k}\) from the encoder after the interleaver are mapped onto symbol \(x_{n,k}\) by the symbol mapper.

In case of CDMA, the complex symbols \(x_{n,k}\) are further spread by a user-distinct spreading code \(u_{n,k} \in \mathbb{C}^{N_u}\) which may depend on symbol index \(n\), if a scrambling code is used. The resulting spread signals are often called “chips” by convention of CDMA systems. In case of IDMA, chips are equivalent to symbols as IDMA applies no spreading codes.

The channel is modeled as an order \(\nu\) finite length impulse response filter with channel taps \(f_{l,k}\) having the normalized energy: 
\[
\sum_{l=0}^{\nu_k} \mathbb{E}\{|f_{l,k}|^2\} = 1,
\]
where \(\mathbb{E}\{\cdot\}\) denotes expectation. The chip time/duration is denoted by \(T_c\). Besides \(\nu_k\), we also consider a user delay \(\tau_k\), which accounts for user-asynchronous transmissions due to imperfect network synchronization. Then, the total delay of user \(k\) becomes \((\tau_k + \nu_k)T_c\).

We define the maximum total delay over all \(K\) users as: 
\[
L_c \triangleq \max_k (\tau_k + \nu_k) \quad \text{and} \quad L_s \triangleq \lceil L_c/N_u \rceil 
\]
In the number of chips and symbols, respectively, where \(\lceil \cdot \rceil\) rounds the argument to the nearest larger integer.

It is convenient to consider an effective channel that is a discrete convolution of the channel taps with the spreading code: 
\[
g_{n,k} \triangleq a_k u_{n,k}^* f_k \in \mathbb{C}^{N_c+2N_u},
\]
where \(a_k \in \mathbb{R}\) is an amplitude reflecting near-far scenarios, \(f_k \triangleq
\( [\mathbf{0}_{\tau_k}^T, f_{0,k}, \ldots, f_{v_{r,k}}, \mathbf{0}_{\tau_{r_2} - \tau_{r_1}}^T] \in \mathbb{C}^{L_2 + 1}, \) is a channel impulse response vector including a user delay \( \tau_k \) and \( \mathbf{u}_{n,k} \triangleq [\mathbf{u}_{n,k}^T, \mathbf{0}_{\tau_{r_2} - \tau_{r_1}}^T] \in \mathbb{C}^{2N_2} \) is the spreading code extended with some zeros; this is simply to make sure that the discrete convolution becomes sufficiently long for the later use. We denote by \( \mathbf{0}_{\tau_k} \) an all-zero column vector of dimension \( \tau_k \) and by \( (.)^T \) matrix transpose.

With the effective channel, the system model can be described in a symbol level, instead of in a chip level. This allows us to work on a unified model that is valid independent of the use of spreading codes. Using the effective channel, the received signal vector \( \mathbf{y}_{n+\ell} \triangleq [y_{n+\ell}N_2, \ldots, y_{n+(\ell+1)N_2-1}]^T \in \mathbb{C}^{N_2} \) may be expressed as:

\[
y_n = \sum_{\ell=0}^{L_s} G(n-\ell, \ell) x_{n-\ell} + \eta_n; \quad \text{where we define a symbol vector:} \quad \mathbf{x}_n \triangleq [x_{n,1}, \ldots, x_{n,K}] \in \mathbb{A}^{K}, \quad \text{a noise vector:} \quad \mathbf{\eta}_n \triangleq [\eta_{n+N_2}, \ldots, \eta_{n+(\ell+1)N_2-1}]^T \in \mathbb{C}^{N_2}, \quad \text{and an effective channel matrix for symbol} \ n \ \text{with delay} \ \ell: \ G(n, \ell) \triangleq [g_1(n, \ell), \ldots, g_K(n, \ell)] \in \mathbb{C}^{N_2 \times K}, \quad \text{where for each user} \ k: \ g_k(n, \ell) \triangleq [g_{n,k}[\ell N_2 + 1], \ldots, g_{n,k}[\ell (1 + N_2)]^T \in \mathbb{C}^{N_2}.
\]

We now develop a time-variant sliding window model for the detection of symbol \( x_{n,k} \). Since there are \( L_s \) pre- and post-cursoirs, the input symbols of \( \mathbf{x}_n \triangleq [x_{n-L_s}, \ldots, \mathbf{x}_n^T, \ldots, x_{n+L_s}] \in \mathbb{A}^{2L_s + 1} \) are taken into account in the sliding window that includes sufficient statistics for the detection of \( x_{n,k} \). Then, we finally get the sliding window model for the detection of symbol \( x_{n,k} \) as:

\[
y_n = \mathbf{H}_n x_n + \eta_n; \quad = 1 \}
\]

We also define a partition of the effective channel matrix:

\[
\mathbf{H}_n \triangleq \begin{bmatrix}
G(n-L_s, L_s) \cdots G(n,0) \\
\vdots & \ddots & \vdots \\
G(n, L_s) \cdots G(n+L_s,0)
\end{bmatrix}.
\]

Then, (1) can be rewritten in the column-wise form:

\[
y_n = \sum_{k=1}^{K} \sum_{\ell=0}^{L_s} h_{n,k} x_{n-k} + \eta_n.
\]

This column-wise channel model will be found useful for describing certain detection algorithms.

**III. ITERATIVE MULTIUSER DETECTION AND DECODING**

The overall receiver structure is depicted in Fig. 2. The receiver for a system of \( K \) simultaneously transmitting users is comprised of a multiuser detector and \( K \) a posteriori probability (APP) decoders that are connected via \( K \) interleavers.

Both multiuser detector and APP decoder are soft-in-soft-out component, which iteratively exchange soft information by accepting and computing log-likelihood ratios (LLRs) as the soft information. According to the turbo principle [7], the output LLRs must be independent of the respective input LLRs. That is ensured by computing the so-called extrinsic LLRs.

The multiuser detector, receives a priori LLRs about code bits sent from the decoder that are defined as:

\[
L_n^m(c_{m,n,k}) = \ln \frac{P\{c_{m,n,k} = +1 | y_n\}}{P\{c_{m,n,k} = -1 | y_n\}},
\]

which is initialized to 0 before the first iteration (i.e., there is no a priori information from the decoder at the beginning: \( c_{m,n,k} \) are equally likely to be 1 and -1). With the a priori LLRs about code bits \( L_m^c(c_{m,n,k}) \) for \( K \) users and the observation \( y_n \), the goal of the multiuser detector is computing the a posteriori LLRs:

\[
L_n^m(c_{m,n,k}) = \ln \frac{P\{c_{m,n,k} = +1 | y_n\}}{P\{c_{m,n,k} = -1 | y_n\}},
\]

and then sending the extrinsic LLRs:

\[
L_n^e(c_{m,n,k}) = L_n^m(c_{m,n,k}) - L_n^m(c_{m,n,k}),
\]

to the decoder after the deinterleaving operation by \( \Pi_k^{-1} \). By using the total probability theory, we can rewrite (6) as:

\[
L_n^e(c_{m,n,k}) = \sum_{x \in \mathbb{A}} \ln \frac{P\{y_n | x_{n,k} = x\} P\{c_{m,n,k} = d_m\}}{P\{c_{m,n,k} = +1\}} + \ln \frac{P\{y_n | x_{n,k} = x\} P\{c_{m,n,k} = d_m\}}{P\{c_{m,n,k} = -1\}},
\]

where \( d_m \in \{+1, -1\} \) denotes the \( m \)-th bit of the symbol \( x \), code bits that are mapped on \( x_{n,k} \) are assumed independent of each other due to the bit interleaver, and

\[
P\{c_{m,n,k} = d_m\} = \frac{1}{2} \left( 1 + d_m \tanh \left( L_n^e(c_{m,n,k})/2 \right) \right).
\]

Each of \( K \) decoders receives the deinterleaver version of the extrinsic LLRs \( L_n^e(c_{m,n,k}) \) from the multiuser detector.
as the channel LLRs $L^d_k (c_{j,k})$ as illustrated in Fig. 2. From the received channel LLRs and according to the code constraints, the decoder computes a posteriori LLRs $L^d_k (c_{j,k})$, which are improved LLRs about the code bits. We do not further discuss the decoding algorithm, since it is the standard function [8].

The channel LLRs $L^d_k (c_{j,k})$ are subtracted from $L^d_k (c_{j,k})$ to get the extrinsic LLRs $L^e_k (c_{j,k})$ (cf. Fig. 2): $L^e_k (c_{j,k}) \triangleq L^d_k (c_{j,k}) - L^d_k (c_{j,k})$, which are sent to the multiuser detector as the new a priori LLRs $L^m_k (c_{m,n,k})$ after being interleaved by $\Pi_k$. In turn, the multiuser detector computes and delivers new LLRs $L^m_k (c_{m,n,k})$. The error performance can improve by some iterations. Finally, the decoder computes a posteriori LLRs about information bits $L^d (b_{i,k})$. Taking its sign: $\hat{b}_{i,k} \triangleq \text{sign} \left( L^d (b_{i,k}) \right)$ gives an estimate of information bit.

Now, we consider three suboptimum linear detectors: MMSE, rake, and soft rake detectors, which are derived for given input a priori information from decoders. These detectors are chosen mainly because of concerns about complexity.

A. Minimum Mean Square Error Detector

In [9], it is shown that the MMSE solution with unbiased constraint adheres the turbo principle without any heuristic adjustment of statistics that is commonly done in the literature. By writing the filter output as:

$$\hat{x}_{n,k} \triangleq \mathbf{w}^H \mathbf{y}_n + \alpha,$$  \hspace{1cm} (9)

where $(\cdot)^H$ denotes Hermitian conjugate transpose and the affine term $\alpha$ is introduced to be jointly optimized with $\mathbf{w}$, the optimization problem may be stated as:

$$\{ \mathbf{w}_{n,k}, \alpha_{n,k} \} \triangleq \arg \min_{\mathbf{w}, \alpha} \ E \left\{ |\hat{x}_{n,k} - x_{n,k}|^2 \right\}.$$  \hspace{1cm} (10)

where the MSE reads as: $\varepsilon \triangleq \mathbb{E} \{ |\hat{x}_{n,k} - x_{n,k}|^2 \}$. The constrained optimization can be solved by using Lagrangian multipliers. The solution can be written as

$$\mathbf{w}_{n,k} \triangleq \frac{C_{n,k}^{-1} h_{n,k}}{h_{n,k} C_{n,k} h_{n,k}} \text{ and } \alpha_{n,k} \triangleq \bar{x}_{n,k} - \mathbf{w}^H n,k \bar{y}_n,$$  \hspace{1cm} (11)

where $\bar{y}_n \triangleq \mathbb{E} \{ y_n \} = H_n \bar{x}_n$, $\bar{x}_n \triangleq \mathbb{E} \{ x_n \}$, and the soft symbol replicas $\bar{x}_n$ are computed as (cf. Eq. 8):

$$\bar{x}_{n,k} \triangleq \mathbb{E} \{ x_{n,k} \} = \sum_{x \in \mathcal{A}} x \sum_{m=1}^{N_n} \mathbb{P} \{ c_{m,n,k} = d_m \}.$$  \hspace{1cm} (12)

We also define

$$C_{n,k} \triangleq C_n - v_{n,k} h_{n,k} \mathbf{H}_{n,k},$$  \hspace{1cm} (13)

$$v_{n,k} \triangleq \mathbb{E} \{ |x_{n,k} - \bar{x}_{n,k}|^2 \} = \sum_{x \in \mathcal{A}} |x|^2 \mathbb{P} \{ x_{n,k} = x \} - |\bar{x}_{n,k}|^2,$$

$$C_n \triangleq \mathbb{E} \{ (y_n - \bar{y}_n)(y_n - \bar{y}_n)^H \} = H_n \mathbf{V}_n \mathbf{H}_n^H + \sigma^2 I.$$  \hspace{1cm} (14)

Here, we assume the noise vector $\eta_n$ to be a zero mean proper complex Gaussian vector with the covariance matrix $\sigma^2 I$ and we assume that transmit symbols are mutually uncorrelated:

$$\mathbf{V}_n \triangleq \mathbb{E} \{ (x_n - \bar{x}_n)(x_n - \bar{x}_n)^H \} = \text{diag} \left\{ v_{n,-L_n}, \ldots, v_n, \ldots, v_{n,+L_n} \right\}.$$  \hspace{1cm} (15)

with $\mathbf{v}_n \triangleq [v_{n,1}, \ldots, v_{n,N_n}]^T$.

From (11), the MMSE estimate in (9) reads as:

$$\hat{x}_{n,k} \triangleq \mathbf{w}_{n,k}^H \mathbf{y}_n,$$  \hspace{1cm} (16)

where the soft interference cancellation is performed:

$$\hat{y}_{n,k} \triangleq y_n - H_n \bar{x}_n + h_{n,k} \bar{x}_{n,k}.$$  \hspace{1cm} (17)

Assuming that the MMSE estimate in (15) is Gaussian distributed, we compute the conditional mean and variance. Then, we end up with the conditional probability density $\phi_{\text{MMSE}}(x)$ in Table I, which is proportional to $p \{ y_n \mid x_{n,k} = x \}$, i.e., $p \{ y_n \mid x_{n,k} = x \} \propto \phi_{\text{MMSE}}(x)$. Therefore, $\phi_{\text{MMSE}}(x)$ can be used for computing the extrinsic LLR in (7).

B. Rake Detector

Rake detector, also known as maximum ratio combining or matched filter, is a conventional single user detection scheme. The rake detector collects the frequency diversity available from multipath channels, but unlike the MMSE detection, it does not explicitly attempt to suppress interference. Thus, the rake detector may suffer from performance degradation in the presence of strong interference as compared to the MMSE detector. However, the rake detector is much simpler than the MMSE detector.

If the rake detector is integrated in the iterative detection and decoding technique, interference suppression can still be performed as the soft interference cancellation by exploiting a priori information from the decoders. Iterative rake detection has been studied, e.g., in [10], [11], [12]. After the soft interference cancellation as in (16), the rake detector tries to maximize the desired signal portion by “being matched” to the channel response:

$$\hat{x}_{n,k}^{(\text{rake})} \triangleq \mathbf{h}_{n,k}^H \hat{y}_{n,k} = \| h_{n,k} \|^2 x_{n,k} + \mathbf{h}_{n,k}^H (\xi_{n,k} - \mathbb{E} \{ \xi_{n,k} \}),$$  \hspace{1cm} (18)

where $\xi_{n,k} \triangleq \sum_{k' \neq k, k' \neq 0} h_{n,k' \neq k} x_{n,k' \neq k} + \eta_n$. Assuming that the residual interference plus noise term $\mathbf{h}_{n,k}^H (\xi_{n,k} - \mathbb{E} \{ \xi_{n,k} \})$ is Gaussian, its probability density can be fully characterized by its mean and variance. Then, we end up with the conditional probability density $\phi_{\text{Rake}}(x)$ in Table I, which can be plugged into (7) for computing the extrinsic LLR.

C. Soft Rake Detector

In the IDMA literature, the so-called soft rake detector has been most frequently considered, e.g. [1], [2], [3], [13], [14], [5]. The soft rake detector is also frequently called elementary signal estimator (ESE). The soft rake detector applies the following approximation in (7):

$$p \{ y_n \mid x_{n,k} = x \} = \prod_{i=1}^{(L_n+1)N_n} p \{ y_{n,i-1} \mid x_{n,k} = x \},$$  \hspace{1cm} (19)

where the elements of the observation vector $y_n$: $y_{n,i} \triangleq e_t^i y_n = e_t^i h_{n,k} \bar{x}_{n,k} + e_t^i \xi_{n,k}$ are assumed to be independent of each other where $e_t$ denotes a $(L_n+1)N_n$ dimensional unit vector whose $i$-th element is 1. Assuming that $e_t^i \xi_{n,k}$ is

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and therefore the proof is omitted. The equivalence can be shown in the previous section can be further extended for system due to, e.g., propagation delays of users. We, however, note that perfect user synchronous transmission is generally difficult to achieve and maintain in practical systems. The equivalence holds not only between the solutions for CDMA systems. However, the MMSE and rake detectors do not require any of the above mentioned complex operations. Using Table I, the three detectors can be conveniently compared.

Table I

<table>
<thead>
<tr>
<th>Detector</th>
<th>Expression</th>
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<tbody>
<tr>
<td>MMSE</td>
<td>$\phi_{\text{MMSE}}(x) \triangleq \exp \left( - \frac{| \mathbf{h} \mathbf{C}^{-1} \mathbf{h}^H \mathbf{x} |^2}{\sigma^2} \right)$</td>
</tr>
<tr>
<td>Rake</td>
<td>$\phi_{\text{Rake}}(x) \triangleq \exp \left( - \frac{| \mathbf{h} \mathbf{C}^{-1} \mathbf{h}^H \mathbf{x} |^2}{\sigma^2} \right)$</td>
</tr>
<tr>
<td>Soft rake</td>
<td>$\phi_{\text{SR}}(x) \triangleq \exp \left( \sum_{i=1}^{(L_s+1)N_u} - \frac{| \mathbf{e}^T (\mathbf{y} - \mathbf{h} \mathbf{x}) |^2}{\sigma^2} \right)$</td>
</tr>
</tbody>
</table>

Gaussian distributed, we compute its mean and variance to get the conditional probability density $\phi_{\text{SR}}(x)$ in Table I, which can be used for computing the extrinsic LLR in (7).

D. Analytical Comparisons of Detectors

This section provides analytical comparisons of the three detectors, which we have derived in the previous sections. Using Table I, the three detectors can be conveniently compared. The MMSE detector is most complex due to matrix inversions $\mathbf{C}^{-1}$, matrix vector multiplications $\mathbf{C}^{-1} \mathbf{h}$, and vector inner products $\mathbf{h}^H \mathbf{x}$. The rake detector is simpler because no matrix inversion has to be computed, but there are still the vector inner products. The simplest among these three detectors is the soft rake, which does not require any of the above mentioned complex operations. Several numerical comparisons of the detectors by means of computer simulations will be provided in Section V. However, under certain conditions, some of the solutions can analytically be shown to be equivalent. In what follows, we prove such equivalence for CDMA and IDMA systems.

1) CDMA: All the detectors generally give different solutions for CDMA systems. However, the MMSE and rake detector are equivalent under the following assumption.

Assumption III.1. K users are perfectly synchronous ($\tau_k = 0, \forall k$) and are transmitting over flat channels ($\nu_k = 0, \forall k$) using orthogonal spreading codes ($K \leq N_u$).

The equivalence is intuitive and straightforward to prove, and therefore the proof is omitted. The equivalence can be summarized as:

$$\phi_{\text{MMSE}}(x) = \phi_{\text{Rake}}(x) \neq \phi_{\text{SR}}(x)$$

(under Assumption III.1.)

We, however, note that perfect user synchronous transmission is generally difficult to achieve and maintain in practical system due to, e.g., propagation delays of users.

2) IDMA: The equivalence of the detectors for CDMA shown in the previous section can be further extended for IDMA systems under a more relaxed condition. Users are not restricted to be synchronous. That might be more relevant in practical systems. The equivalence holds not only between the MMSE and rake detectors, but also for the soft rake detector. Now, we make the following assumption in this section:

Assumption III.2. K asynchronous users are transmitting over flat channels ($\nu_k = 0, \forall k$).

Under Assumption III.2, the effective channel matrix $\mathbf{H}$ in (2) has dimension $(L_s + 1) \times (2L_s + 1)K$ where $L_s = L_c = \text{max}_k \tau_k$ because IDMA does not apply spreading code, i.e., $N_u = 1$ and symbol delay is equivalent to chip delay. The effective channel vector can be written as: $\mathbf{h}_{n,k} = [0^T \mathbf{f}_{0,k} \ 0^T \mathbf{f}_{L_s - \tau_k}]^T = \mathbf{f}_{0,k} \mathbf{e}_{\tau_k + 1}$. Other effective channel vectors $\mathbf{h}_{n,k}$ of $\mathbf{H}$ are simply time shifted versions of $\mathbf{h}_{n,k}$ and thus, they have at most one non-zero element $\mathbf{f}_{0,k}$. Therefore, the covariance matrix $\mathbf{C}_n = \sum_{k=1}^{K} \sum_{\ell=-L_s}^{L_s} \mathbf{v}_{n-\ell,k} \mathbf{h}_{n-\ell,k} \mathbf{h}_{n-\ell,k}^H + \sigma^2 \mathbf{I}$ has non-zero elements only in its main diagonal. Then,

$$\mathbf{C}_{n,k} = \mathbf{C}_n - \mathbf{v}_{n,k} \mathbf{h}_{n,k} \mathbf{h}_{n,k}^H = \mathbf{C}_n - \mathbf{v}_{n,k} \mathbf{f}_{0,k}^2 \mathbf{e}_{\tau_k + 1} \mathbf{e}_{\tau_k + 1}^H,$$

is diagonal as well. With the diagonal structure, we get the following equalities:

$$\mathbf{h}_{n,k}^\mathbf{H} \mathbf{C}_{n,k}^{-1} \mathbf{h}_{n,k} = \mathbf{f}_{0,k}^2 \mathbf{e}_{\tau_k + 1}^T \mathbf{e}_{\tau_k + 1},$$

$$\mathbf{C}_{n,k}^{-1} \mathbf{h}_{n,k} = \mathbf{f}_{0,k} \mathbf{e}_{\tau_k + 1}^T (\mathbf{e}_{\tau_k + 1} \mathbf{C}_{n,k} \mathbf{e}_{\tau_k + 1})^{-1} \mathbf{e}_{\tau_k + 1},$$

Using these equalities, it can be shown that:

$$\phi_{\text{MMSE}}(x) = \phi_{\text{Rake}}(x) = \phi_{\text{SR}}(x)$$

(under Assumption III.2.)

This equivalence may also be applicable for broad band transmissions using orthogonal frequency division multiplexing (OFDM) that transforms broad band signal into a set of narrow band signals. Several references can be found that combine IDMA with OFDM modulation [15], [16]. It should be pointed out that the equivalence guarantees the MMSE solution without matrix inversions or matrix vector multiplications unlike CDMA, which requires the MMSE detector even on a flat channel, if users are asynchronous.

IV. COMPLEXITY ANALYSIS OF MMSE DETECTOR FOR CDMA AND IDMA

The complexity of the iterative MMSE detector arises from the fact that the MMSE filter has to be computed for $N_u$ symbols of $K$ users that is repeated over $Q$ iterations. There have been a number of complexity reduction techniques in the literature, e.g., [17], [18], [19]. In this section, we do not discuss particular complexity reduction techniques. Instead, we focus on essential difference in complexity between CDMA and IDMA.

For notational convenience, we introduce the following definitions:

$$M \triangleq (L_s + 1)N_u \quad \text{and} \quad N \triangleq (2L_s + 1)K. \quad (18)$$

Then, for example, the dimensions of the channel matrix and covariance matrix can be rewritten as

$$\mathbf{H}_n \in \mathbb{C}^{M \times N} \quad \text{and} \quad \mathbf{C}_{n,k} \in \mathbb{C}^{M \times M}.$$
The complexity is compared under the assumption that both systems operate with the common parameters: number of users $K$, number of information bits per transmission block $N_u$, modulation alphabet $A$, bandwidth, and chip delay $L_c$ (computed from common channel memories $r_k$ and user delays $\tau_k$). With these common parameters, we end up with different values of $M$, $N$, and $N_s$, which are the three main factors determining the complexity.

Let us start with a comparison of the number of rows of channel matrix $M \triangleq (L_s + 1)N_u$. Since $L_s \triangleq \lceil L_c/N_u \rceil$, it holds that:

$$M^{(\text{CDMA})} \triangleq (L_s + 1)N_u \geq (L_c/N_u + 1)N_u \geq L_c + 1 \triangleq M^{(\text{IDMA})},$$

with equality for $N_u = 1$. Thus, the number of rows of channel matrix for CDMA is larger than for IDMA. For the number of columns of channel matrix $N \triangleq (2L_s + 1)K$, one can write

$$N^{(\text{IDMA})}/N^{(\text{CDMA})} = (2L_c + 1)/(2L_s + 1) \sim N_u \quad \text{for large } L_c.$$

Thus, the number of columns of channel matrix for IDMA is generally much larger than for CDMA. We note that the difference of $N$ grows much faster than that of $M$ with respect to the spreading factor $N_u$.

Now, we compare the number of symbols $N_s$. Since bandwidth expansion for IDMA is fully exploited by low rate code before the bit-interleaver, we can write:

$$N_s^{(\text{IDMA})} = N_u \cdot N_s^{(\text{CDMA})}.$$

Since we need MMSE filters for $N_s$ symbols, IDMA has to compute $N_s$ times larger number of filters than CDMA.

From the analysis above, we see that the MMSE detection for IDMA may be much more complex than for CDMA. As we will observe later, IDMA generally performs well with the simpler soft rake detector in many scenarios. But, in scenarios where the MMSE detector should be used rather than the soft rake to get better performance, IDMA can be more complex than CDMA. Nevertheless, when we recall the symbol-wise complexity reduction technique according to the time-average of covariance matrix [17], [18], the matrix inversion have to be computed only once over symbols, and then the number of symbols $N_s$ is not the main complexity factor anymore. This technique can be exploited over time-invariant channels.

In the presence of user asynchronism, however, CDMA either does not perform well without using user-distinct scrambling codes, or cannot use the time-average technique due to the use of scrambling sequences, which effectively make the time-invariant channel vary over time. Therefore, IDMA may be more likely to gain tremendous complexity reductions by the time-invariant approach without compromising the performance as compared to CDMA, when channel is quasi static. If channel varies quickly within each transmission block, then the time-invariant approach may no be applied. Then, the MMSE detector for IDMA is much more complex than for CDMA.

This section presents comparisons of CDMA and IDMA by means of computer simulations. The simulation parameters are summarized in Table II. We note that, when the number of users, $K$, exceeds the spreading factor $N_u$, the common short code $[+1, -1, +1, -1]$ is multiplied with user distinct long scrambling sequences according to [20] to get individual spreading codes, since orthogonal short codes cannot be assigned to all users of CDMA.

### V. Performance Analysis of CDMA and IDMA

We start with the performance comparisons of CDMA and IDMA for $K = 6, 8$ users on an AWGN channel. Fig. 3 illustrates the BER performance after 10 iterations. The results are obtained by using the MMSE detector for CDMA to avoid the performance degradation by other two detectors, whereas for IDMA we use the soft rake detector, which is equivalent to other detectors on an AWGN channel (cf. Section III-D for the analytical proof). For both CDMA and IDMA, we see that the performance approaches the single user bound above certain $E_b/N_0$ values, but the performance of IDMA is superior to that of CDMA and the performance gap grows as the number of users increases. Although not presented here, we note that the difference becomes even greater if the simpler rake or soft rake detector is used for CDMA, instead of the MMSE.

#### A. AWGN Channel

We start with the performance comparisons of CDMA and IDMA for $K = 6, 8$ users on an AWGN channel. Fig. 3 illustrates the BER performance after 10 iterations. The results are obtained by using the MMSE detector for CDMA to avoid the performance degradation by other two detectors, whereas for IDMA we use the soft rake detector, which is equivalent to other detectors on an AWGN channel (cf. Section III-D for the analytical proof). For both CDMA and IDMA, we see that the performance approaches the single user bound above certain $E_b/N_0$ values, but the performance of IDMA is superior to that of CDMA and the performance gap grows as the number of users increases. Although not presented here, we note that the difference becomes even greater if the simpler rake or soft rake detector is used for CDMA, instead of the MMSE.

#### B. Multipath Channels

Next, we evaluate the performance over multipath channels. We study first the performance comparison of different detec-
matters for CDMA and IDMA systems on the 5 paths complex-valued fixed channel defined by Porat in [21]: $f_{\text{Porat}} = [0.49 + j0.10, 0.36 + j0.44, 0.24, 0.29 - j0.32, 0.19 + j0.39]$, which is normalized to have unit norm. This channel has poor spectral characteristics. Fig. 4 shows the BER performance averaged over $K = 4$ users after 10 iterations. The performance of single user transmission over an AWGN channel is also included as a comparison. In case of CDMA, the performance of the soft rake and rake detectors is poor. The performance significantly improves by using the MMSE detector, but it does not approach the single user bound on an AWGN channel. In contrast to CDMA, the performance of IDMA approaches the single user bound on an AWGN channel using any of the detectors, but with different convergence behaviors. Recall that all the detectors are equivalent for IDMA on an AWGN channel (or more generally on frequency flat channels), and now we observe the different performance on the multipath channel. The MMSE detector demands the most complex operations and results in the best performance. The rake detector is less complex and performs worse than the MMSE. The performance is further degraded with the least-complex soft rake detector.

We note that the results presented here are obtained from the particular choice of channel defined and we should not put too much emphasis on the consequence. Nevertheless, we see the robustness of IDMA against CDMA on that channel having poor spectral characteristics. The choice of detectors makes difference on the performance, but the impact is less significant for IDMA as compared to CDMA.

Fig. 5 shows the BER performance over randomly generated 16 paths ($\tau_k = 15, \forall k$) channels following the uniform delay profile. It can be observed in Fig. 5 that there is performance degradation for CDMA against IDMA for $K = 8$ users that is not present for $K = 4$ users. We also observe that the soft rake detector does not perform as well as the MMSE detector anymore for IDMA.

B. Analysis Using Extrinsic Information Transfer Charts

We analyze the system using extrinsic information transfer (EXIT) chart [6], which is a convenient tool for studying iterative processing techniques. The EXIT chart shows the average mutual information between the transmitted code bits and the input LLRs, which are obtained from the other soft-in soft-out block, versus the average mutual information between the transmitted code bits and the extrinsic output LLRs, which are sent to the other soft-in soft-out block.

In our system, the horizontal axis is the mutual information $I(c', L^m_w(c'))$ at the input of the multiuser detector, which is also the mutual information at the output of the decoder $I(c, L^d_w(e))$ (cf. Fig. 2 and (6)). The vertical axis is the mutual information $I(c', L^m_w(c'))$ at the output of the multiuser detector, which is also the mutual information at the input of the decoder $I(c, L^d_w(e))$. Curves for the relation between input and output mutual information can be plotted for both soft-in soft-out blocks: the multiuser detector and the decoder in our system. The trajectory between the two curves starting from the multiuser detector without a priori information, i.e., $I(c', L^m_w(c')) = I(c, L^d_w(e)) = 0$, illustrates the performance improvements by iterations. The average mutual information between code bits $c$ and LLRs $L(e)$ can be computed by numerical evaluation of [6]: $I(c, L(e)) = \sum_{c' = +1,-1} \int_{-\infty}^{+\infty} p(L(e) | c) \log_2 \left( \frac{p(L(e) | c') + p(L(e) | c)}{2p[L(e)|c]} \right) dL(e)$. For the EXIT chart computation, it is assumed that the input LLRs are statistically independent and Gaussian distributed.

AWGN Channel

Fig. 6 illustrates the EXIT curves of the decoders and the detectors for CDMA and IDMA systems with $K = 4$ or 8 users over an AWGN channel at $E_b/N_0 = 5$ dB. These EXIT curves correspond to the BER performance in Fig. 3.

Two EXIT curves are plotted for the decoder: only with the rate $R_c = 1/2$ convolutional code for CDMA and additionally with the rate $R_c = 1/4$ repetition code for IDMA. Since vertical and horizontal axes are, respectively, input and output mutual information for the decoder, it can be observed that higher output mutual information $I(c, L^d_w(e))$ for a given input $I(c, L^d_w(e))$ is obtained by employing the additional repetition code. We note that EXIT curves of the decoder are independent.
BER performance results in Fig. 3 at $E_s/N_0$ the single user bound is not possible. That agrees with the tunnel by using any detector for CDMA and convergence to $K$ user bound among the three detectors (at the price of larger detector using the MMSE detector. Thus, the MMSE detector tunnel opened between the EXIT curves of the decoder and the output LLRs of the detector as well.

In order for the iterative process to reach the single user bound, there must be an open tunnel between the EXIT curves of detector and decoder. The convergence behavior is then determined by the width of the tunnel; narrower the tunnel is, larger the number of iterations must be.

For CDMA, it can be observed in Fig. 6 that, for a given input mutual information $I(c', L_c^m(c'))$, the MMSE detector results in much higher mutual information $I(c', L_c^m(c'))$ than the other two detectors. With $K = 4$ users, there is the widest tunnel opened between the EXIT curves of the decoder and the detector using the MMSE detector. Thus, the MMSE detector requires the least number of iterations to end up with the single user bound among the three detectors (at the price of larger complexity per iteration). With $K = 8$ users, there is no open tunnel by using any detector for CDMA and convergence to the single user bound is not possible. That agrees with the BER performance results in Fig. 3 at $E_b/N_0 = 5$ dB.

In case of the IDMA system, there are open tunnels for both cases of $K = 4$ and 8 users. In the latter case, although the tunnel is narrow, the single user bound should be reached after a sufficient number of iterations. That can be also evidenced from the BER performance at $E_b/N_0 = 5$ dB in Fig. 3.

In this paper, we performed comprehensive comparisons of IDMA and CDMA systems. Three suboptimum iterative linear detectors have been considered: the MMSE, rake, and soft rake detectors from practical complexity concerns. The three detectors were analytically shown to be equivalent for IDMA over flat channels without computationally expensive matrix inversions and matrix vector multiplications. This is valid in cases of $K = 4, 8$ users over an AWGN channel at $E_b/N_0 = 5$ dB and EXIT curves of decoders (cf. Fig. 3 for the corresponding BER performance).

Multipath Channel

Fig. 7 illustrates the EXIT curves of the decoders and the detectors for the CDMA and IDMA systems with $K = 4$ users on the Porat channel at $E_b/N_0 = 5$ dB. These EXIT curves correspond to the BER performance in Fig. 4. In contrast to the previous case of an AWGN channel, the EXIT curves of the IDMA system are now computed not only for the soft rake detector but also for the rake and MMSE detectors, as their performance can be different on a multipath channel.

For the CDMA system, the tunnel is open between the EXIT curves of the detector and decoder only when the MMSE detector is used. Convergence to the single user bound is not possible with the other two detectors. That can be also observed from the BER performance in Fig. 4 at $E_b/N_0 = 5$ dB.

For the IDMA system, the MMSE detector yields better mutual information than the other two detectors. Nevertheless, the tunnel between the EXIT curves of the decoder and detector is open by using any of the detectors. That means, using any detector can reach the single user bound after several iterations, although a different number of iterations is necessary for each detector. However, the BER performance using the soft rake detector in Fig. 4 at $E_b/N_0 = 5$ dB does not converge to the single user bound. This is due to the limited block size. Although it is not shown in this paper due to space limitation, the BER performance approaches the single user bound using the soft rake detector by increasing the block size, e.g., in factor four.

VI. CONCLUSIONS

In this paper, we performed comprehensive comparisons of IDMA and CDMA systems. Three suboptimum iterative linear detectors have been considered: the MMSE, rake, and soft rake detectors from practical complexity concerns. The three detectors were analytically shown to be equivalent for IDMA over flat channels for asynchronous users. It was then pointed out that the equivalence guarantees the MMSE solution for IDMA over flat channels without computationally expensive matrix inversions and matrix vector multiplications. This is
not the case for CDMA system in general since CDMA is sensitive to user asynchronism.

We performed computer simulations in various scenarios. In case of CDMA, it was observed that the performance degradation is severe by the rake and soft rake detectors as compared to the MMSE detector in most scenarios. For IDMA, the simplest soft rake detector performs nearly as well as the most complex MMSE detector in many scenarios except highly user-loaded scenarios or for a channel with poor frequency characteristic.

We analyzed the system using EXIT charts, which revealed predictions of iterative processing behavior assuming a sufficiently large block size. The EXIT chart analysis confirmed the superiority of IDMA against CDMA and also helped predict the impact of block size on the performance. In highly user-loaded scenarios or for channel with poor spectral characteristics, the soft rake detector for IDMA does not perform well anymore and/or requires a large number of iterations. In the latter case, a large block size is necessary to avoid increasing correlations in extrinsic information over iterations. That may limit its applicability to delay sensitive applications where short block size is used. In either case, there are clear advantages of employing the MMSE detector, which is however more complex and one has to consider complexity reduction techniques, e.g., [17], [18], [19].

Instead of discussing such particular complexity reduction techniques, we focused on essential difference in complexity between CDMA and IDMA, when using the MMSE detector. By comparing the number of symbols and the dimension of covariance matrix to be inverted, we observed that the MMSE detection for IDMA may be considerably more complex than for CDMA. However, if we exploit the symbol-wise complexity reduction technique according to the time-average of covariance matrix [17] over time-invariant channels, matrix inversion has to be computed only once over symbols, and then the number of symbols is not the main complexity factor anymore. In the presence of user asynchronism, however, CDMA either does not perform well without using user-distinct scrambling codes, or cannot use the time-average technique due to the use of scrambling sequences, which effectively make time-invariant channel vary over time. Therefore, IDMA may be more likely to gain tremendous complexity reductions by the time-invariant approach without compromising the performance as compared to CDMA, when channel is quasi static.

Although it was not discussed in this paper due to space limitation, there are other possible advantages of IDMA that exploit the typical low rate code construction rule of concatenating repetition code and convolutional code [22], [23].

References