Investigating Student Learning of the Voltage and Potential Concepts in Introductory Electrical Engineering

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Abstract—Voltage is one of the most fundamental concepts in electrical engineering, but nevertheless has been shown to be a difficult concept for many students. To help address these difficulties, we designed a tutorial worksheet similar to those published by the Physics Education Group at the University of Washington. In this activity, students were introduced to electric potential and compared and contrasted it to voltage. As electric potential and Kirchhoff’s Voltage Law are closely related, we assumed learning about potential would help students gain a better qualitative understanding of voltage. Post-test and exam data from an introductory circuits course show that after the tutorial many students still had difficulties with voltage and potential. More than three quarters of the responses concerning voltage and potential were inconsistent. This suggests that most students were unable to link those two concepts. We therefore conclude that potential and voltage remain conceptually very different for students in their learning process. As tutorial worksheets have proved to be very effective in students overcoming conceptual difficulties, it is likely that there are specific difficulties with these concepts that have not yet been identified.

Keywords—student understanding, voltage, potential, tutorial worksheet, concept, student learning.

I. INTRODUCTION

Voltage is one of the most important concepts in electrical engineering. The concept of voltage is closely related to the concept of electric potential, with any voltage being the difference between two potentials: \( V_{XY} = \phi_X - \phi_Y \). Thus, wherever one of these concepts occurs, it can be replaced by the other one. It can be argued, that both concepts are just different descriptions of the same underlying physical entity.

While there is virtually no course on electrical engineering, in which voltage is not used, many lectures and curricula spend very little time on the electric potential. Often, it is simply defined by its relation to voltage and not examined further. However, potential has the advantage that its theoretical description is much easier than that of voltage. It is a scalar defined for every point in the circuit instead of a scalar defined between every pair of points. This makes potential easier to visualize in a diagram. For example, it is possible to use color coding to indicate different potentials in a circuit, as for example students are asked to do in task 2.1.c. in Fig. 1. This is impossible for voltages. Additionally, it is rather easy to compare the electric potential to other potential fields, e.g. the gravitational potential, which is familiar to many students.

Many students have difficulties with the conceptual understanding of voltage [1]. Investigations at our institution have confirmed these difficulties to be present also among our students [2], [3]. In a course where potential is usually not introduced at all, we decided to spend considerable time on that concept. We hoped that through its close relation to voltage, an understanding of potential would also further students’ understanding of voltage.

For the introduction of potential, we chose to use tutorial worksheets, as introduced by the Physics Education Group at the University of Washington [4]. These are structured worksheets for collaborative group work that focus on conceptual questions and are designed to address known student difficulties. In our group, many tutorial worksheets have been developed based on our own research. In addition, we have gained considerable experience in implementing such tutorial worksheets [2], [5].

In the following section, we will describe the course in which this investigation was conducted, as well as our methodology. Afterwards, the specific materials used (i.e. the tutorial worksheet, post-test, and relevant exam question) will be described. We will give an overview over the students’ answers in the post-test and exam and highlight important results. Finally, we will explain our interpretation of these results and draw conclusions.

II. CONTEXT OF THE INVESTIGATION

The investigation was carried out in an introductory level electrical engineering course for mechanical, process, and logistics engineering students, most of whom were in their first semester. The course is organized and given by faculty from another department at our university. It is taught in a traditional manner with 90 minutes of lecture and 45 minutes of recitation sessions per week. The former is attended by about 350 students, while the latter are split into groups of about 20 students. The course covers the subject of circuit theory with direct and alternating, as well as three-phase current. Potential is not part of the curriculum.

For several years, we have implemented four tutorial worksheets in this course, two of which are done in the recitation sessions and two during lecture time with all students and about 15 teaching assistants present. The two worksheets in the recitation sessions are based on hands-on observations.
that students make in the context of batteries and light bulbs. The course mostly focuses on quantitative analysis of circuits, while the worksheets focus on qualitative understanding. On each exam, at least one question focuses on the concepts in qualitative reasoning developed in the tutorials. All exam questions are in the multiple choice format.

In our research, we mainly investigate conceptual understanding of students, as in our opinion, this is required to reliably apply one’s knowledge. To reach this kind of understanding, students have to develop concepts for themselves, ideally guided by an instructor and other learning aids. We believe that tutorial worksheets are a very good method to accomplish this aim. Whenever possible, we base the development of our instructional materials on empirical evidence and quantitative data.

### III. Materials Used

This section illustrates three of our contributions to the course described above: the third tutorial worksheet, which introduced the concept of potential; the post-test to evaluate this tutorial; and one exam question that we designed and administered.

The goal of the third tutorial worksheet was to improve students’ understanding of voltage and Kirchhoff’s Voltage Law (KVL) through the introduction of electric potential. Most parts of the worksheet were adapted from two other tutorials. The first page revisits the properties of parallel and series connections and their implications for current and voltage. It is based on questions from a tutorial that introduces circuit analysis [6]. The next one and a half pages define and introduce the electric potential and investigate its properties. They are based on a whole worksheet on potential difference [7]. The rest of the tutorial examines the relationship between elements in a circuit and equations used to describe that circuit. This part is again based on the worksheet about circuit analysis [6]. Fig. 1 shows an English translation of the second page of the tutorial.

This tutorial was used in a large lecture hall during lecture time. To allow teaching assistants to reach everyone, students were asked to only occupy every other row of seats. Also, students were asked to work in groups of three and work was structured in segments of about fifteen minutes. After each segment, a lecturer summarized some but not all answers to questions of the current segment. This structure ensured that all students could be made aware of the main points of the worksheet.

During the 90-minute tutorial session, two and a half pages of the tutorial were covered. The students worked through all exercises concerning potential, but only started on those concerning the relation between circuit diagrams and equations.

Seven weeks later, at the beginning of the next tutorial session, a two-page post-test was given. The first page of this post-test asked questions about voltage and potential. An English translation of the first two questions is shown in Fig. 2. Students were given 12 minutes for the post-test. The first two questions ask students to rank voltages and potentials in a given circuit and to give an explanation for their ranking in each case.

About two months later, the exam was given. All questions were multiple choice questions and students were not asked to explain their reasoning. A circuit from the first of eight exam questions is shown in Fig. 3.

### IV. Results

In the post-test and the exam, we observed two things: The answers concerning voltage and potential often contradicted each other. Also, the majority of students gave answers that violated KVL.

The post-test shown in Fig. 2 was administered to 311 students. Among their responses were 27 distinct rankings of voltages and 47 distinct rankings of potentials. Table I gives an overview of the most prominent rankings. The correct answers, printed in bold face, were given by only 3% and 1% of the students, respectively.

Of the 219 students who ranked both quantities, 84% gave inconsistent answers. Their ranking of the potentials contradicted their voltage ranking. For example, many students correctly stated $V_{AB} > V_{AM}$, but then answered that potentials D and B were equal ($\phi_D = \phi_B$).
The circuit at right contains two identical batteries, which can be treated as ideal voltage sources. The short line indicates the negative clamp of the battery, the long line the positive clamp. The symbol at node D indicates that the potential there is zero. Bulbs 1 and 2 are also identical, switch S is open at first.

When the switch is open:

1. Rank the three voltages $V_{AD}$, $V_{AX}$, $V_{AB}$ by their absolute value. Please use the relational operators $>$, $<$ and $\neq$. Please indicate, if two voltages are equal or a voltage is equal to zero.

   **Answer:**

   **Explanation:**

2. Rank nodes A through D by their electric potential $\phi_a$ through $\phi_d$.

   **Answer:**

   **Explanation:**

Fig. 2. Translation of the first part of the post-test.

**TABLE I. OVERVIEW OF MOST COMMON AND CORRECT ANSWERS TO THE POST-TEST SHOWN IN Fig. 2.**

<table>
<thead>
<tr>
<th>Voltage Ranking $^1$</th>
<th>Quantity</th>
<th>Potential Ranking $^2$</th>
<th>Quantity</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_{AC} &gt; V_{AD} &gt; V_{AB}$</td>
<td>2 %</td>
<td>$\phi_a &gt; \phi_d &gt; \phi_c &gt; \phi_b$</td>
<td>1 %</td>
</tr>
<tr>
<td>$V_{AC} &gt; V_{AD} &gt; V_{AB}$</td>
<td>1 %</td>
<td>$\phi_a &gt; \phi_d = \phi_c = \phi_b$</td>
<td>18 %</td>
</tr>
<tr>
<td>$V_{AD} &gt; V_{AB} = V_{AC}$</td>
<td>40 %</td>
<td>$\phi_a &gt; \phi_d &gt; \phi_c &gt; \phi_b$</td>
<td>9 %</td>
</tr>
<tr>
<td>$V_{AD} &gt; V_{AB} = V_{AC}$</td>
<td>19 %</td>
<td>$\phi_a &gt; \phi_d &gt; \phi_c &gt; \phi_b$</td>
<td>5 %</td>
</tr>
<tr>
<td>$V_{AB} &gt; V_{AD} &gt; V_{AC}$</td>
<td>5 %</td>
<td>$\phi_a &gt; \phi_d &gt; \phi_c &gt; \phi_b$</td>
<td>5 %</td>
</tr>
<tr>
<td>Blank</td>
<td>2 %</td>
<td>Blank</td>
<td>15 %</td>
</tr>
</tbody>
</table>

The first three tasks in the exam asked students to compare the brightness of certain bulbs in the circuit in Fig. 3. In the subsequent five tasks, students were asked if the absolute values of certain quantities would (a) increase, (b) stay the same, (c) decrease to 0, or (d) decrease, but not to 0, when bulb D was removed from its socket. The quantities considered were the voltage between X and Y ($V_{XY}$), the voltage between Y and Z ($V_{YZ}$), and the currents $I_i$, $I_1$, and $I_3$. The students’ answers to these five questions can be found in Table II with correct answers indicated in bold.

Instead of looking at individual answers to tasks in the exam, we will examine the consistency of students’ answers with KVL: One closed loop in the circuit from Fig. 3 is across bulb C, bulb D, and the battery. The text in the exam states that the battery can be treated as an ideal voltage source. Table III(a) shows the distribution of answers to the questions about the changes to $|V_{XY}|$ (task 4) and $|V_{YZ}|$ (task 5) when bulb D is removed. The battery voltage stays the same when the bulb is removed. From KVL, it follows that the sum of the voltages $V_{XY}$ and $V_{YZ}$ also has to stay the same. For the underlined combinations of answers, $V_{XY} + V_{YZ}$ cannot stay the same, as e.g. both quantities decrease. 70 % of the students gave such an answer that is inconsistent with KVL.

We will now contrast this by examining the consistency of students’ answers with Kirchoff’s Current Law (KCL).

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$^1$Most students did not use absolute value signs in their responses. As all three voltages are positive, the absolute value signs are not printed here.

$^2$Correct answers indicated by bold font.

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The battery in the circuit at right can be treated as an ideal voltage source. The long line indicates the positive terminal. The five bulbs are identical. All occurring voltages are within operating range of the bulbs.

**TABLE II. ANSWERS TO SELECTED QUESTIONS IN THE EXAM ABOUT THE CHANGES TO QUANTITIES AFTER REMOVING BULB D.**

<table>
<thead>
<tr>
<th>Task</th>
<th>Quantity</th>
<th>Answer $^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>$</td>
<td>V_{XY}</td>
</tr>
<tr>
<td>5</td>
<td>$</td>
<td>V_{YZ}</td>
</tr>
<tr>
<td>6</td>
<td>$</td>
<td>I</td>
</tr>
<tr>
<td>7</td>
<td>$</td>
<td>I_1</td>
</tr>
<tr>
<td>8</td>
<td>$</td>
<td>I_3</td>
</tr>
</tbody>
</table>

**TABLE III. RELATIVE DISTRIBUTION FOR PAIRS OF ANSWERS TO QUESTIONS FROM THE TASKS IN TABLE II.**

(a) For all 483 students who answered both tasks. Distractors as shown in Table II.

<table>
<thead>
<tr>
<th>Task 5 ($V_{XY}$)</th>
<th>Task 4 ($V_{YZ}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>a</strong> $2 % 2 %$</td>
<td>a 2 % 2 % 14 % $2 %$</td>
</tr>
<tr>
<td><strong>b</strong> $2 % 57 %$</td>
<td>b 57 % 2 % 14 % $2 %$</td>
</tr>
<tr>
<td><strong>c</strong> $5 % 9 %$</td>
<td>c 5 % 9 % 1 % 2 %</td>
</tr>
<tr>
<td><strong>d</strong> $0 % 17 %$</td>
<td>d 17 % 0 % 1 % 2 %</td>
</tr>
</tbody>
</table>

Combinations conflicting KVL: 70 %

(b) For all 196 students who answered tasks 7 and 8 and chose a in task 6. Distractors as shown in Table II.

<table>
<thead>
<tr>
<th>Task 8 ($I_1$)</th>
<th>Task 7 ($I_2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>a</strong> $59 % 2 %$</td>
<td>a $59 % 2 % 2 %$</td>
</tr>
<tr>
<td><strong>b</strong> $12 % 13 %$</td>
<td>b $12 % 13 % 1 % 2 %$</td>
</tr>
<tr>
<td><strong>c</strong> $0 % 0 %$</td>
<td>c $0 % 0 % 1 % 2 %$</td>
</tr>
<tr>
<td><strong>d</strong> $2 % 17 %$</td>
<td>d $2 % 17 % 1 % 4 %$</td>
</tr>
</tbody>
</table>

Combinations conflicting KCL: 21 %

Specifically, we will look at those 41 % of the students that indicated in task 6 that the current $I$ through the battery remains the same. While this answer is incorrect and indicates a previously identified misconception [8], it may or may not lead to answers consistent with KCL. Table III(b) shows the distribution of these students’ answers to the questions about $I_1$ (task 7) and $I_3$ (task 8). During the exam, a clarifying announcement was made that removing bulb D would result in the wire between points Y and Z to be disconnected. As these students indicated that $I$ remains the same and $I_2$ has to decrease to zero because of the wire being disconnected, the sum of $I_1$ and $I_3$ has to increase. For the underlined combinations of answers in III(a) this cannot be true, as neither of the two currents is indicated to increase. In all, only 21 % of the students that assumed the battery current to be constant gave answers that definitely contradict KCL.

Not many students gave explanations to their answers in the post-test, especially on the later questions. However, several questions in the post-test and exam have been given in similar form in tests and exams in other courses. The answers from students in this course were in general similar to the answers given in previous years. Thus, we do believe the students in this year’s course were fairly typical students.

We did expect many students to have trouble with ranking the voltages and potentials in the post-test because of the open circuit (open switch) between points B and C. Several faculty
members at our university have mentioned open circuits to be problematic for students, and data from different publications support this assumption. In the DIRECT test, a concept inventory on DC circuits, one of the most difficult questions is question 28 (version 1.0), which asks about the voltage across an open switch [9]. Hussain also reports that less than 5% of students were able to correctly describe the voltage across an open switch [10]. For the same reason, we expected task 5 ($V_{Z}$) in the exam to be troublesome for many students. This proved to be true.

V. CONCLUSION

The results from the post-test and exam show that after instruction the majority of students in this course did not correctly apply KVL in qualitative questions. As KVL is a direct consequence of the electric potential, these students also cannot have a functional understanding of the electric potential.

Of those students that incorrectly treated the battery in the exam as a current source, about four fifths gave answers that are consistent with KCL. From the fact that so many students with a common misconception did not give answers that contradicted KCL, we conclude that the application of a conservation law was not problematic for students. Instead, the recognition that such laws hold for the specific quantities of voltage and/or potential proved to be difficult. If they had understood “the physics”, they would have been able to do “the math”.

The post-test showed that virtually no student had a functional understanding of potential in a test that most instructors would judge fair. Additionally, the conflicting answers to the current and voltage ranking questions in the post-test indicated that students did not see the relationship between voltage and electric potential. Instead of recognizing both concepts as different representations of the same underlying physical entity, and thus getting a better understanding of it, potential was just another independent concept they had to learn.

In papers by our group and others, such as the Physics Education Group at the University of Washington, the tutorial approach has been shown to be quite successful in a variety of topics and settings [2], [5], [11]. Members of our group have been involved in the development and use of tutorial worksheets for more than ten years. The fact that our attempt to help students with their understanding of voltage and potential has failed so dramatically therefore requires a different explanation than worksheets being an ineffective method for instruction.

Questions concerning an open switch might be a good task to evaluate students’ understanding of voltage and potential. While it is commonly known to be troublesome for students, there seems not to be much research on student understanding of open switches. It has been suggested, that students incorrectly apply Ohm’s Law for switches [10], [13], but the problem might also be that students see voltage (or potential) as a property of current. Thus, without current, there could be no voltage (or potential).

As there were almost no students that correctly ranked the potentials in the post-test, we could not see if a functional understanding of potential also helped the students in understanding the concept of voltage. Besides a better worksheet, a less challenging set of tasks about the two quantities would help to investigate this further. Moreover, our results suggest, that there are specific difficulties with these concepts that have not yet been identified. An in-depth investigation possibly using student interviews might help to uncover such difficulties.

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REFERENCES


